

# A Mathematical Analysis of Oregon Lottery Keno

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## Introduction

This report provides a detailed mathematical analysis of the keno game offered through the Oregon Lottery (<http://www.oregonlottery.org/games/draw-games/keno>), hereafter called Oregon Lottery Keno. Oregon Lottery Keno is available at restaurants, bars, and other locations throughout the state of Oregon (United States). It is a traditional 80-number keno game with two pay tables, an optional award multiplier, an optional “Bulls-Eye” side bet, and a progressive jackpot. A new game is played every four minutes. Players may wager fixed amounts between \$1 and \$20 per game and may play up to 100 consecutive games on a single ticket. Oregon Lottery Keno has a payout limit of \$1,000,000 per ticket per game.

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## Game Overview

A blank Oregon Lottery Keno game slip is shown in Figure 1. The player makes his or her selections on the front of the game slip. The back of the game slip shows the available pay tables, along with other relevant information. A separate brochure explains how to play keno.

**KENO**  
EVERY 4 MINUTES

1 Choose how many spots you want to play.  
1 2 3 4 5 6 7 8 9 10

2 How much do you want to wager per game?  
\$1 \$2 \$3 \$4 \$5 \$10 \$20

3 How many consecutive games do you want to play?  
1 2 3 4 5 10 15 20 50 100

4 **Special Keno Option**  
Offers larger top prizes but fewer smaller prizes per game. No extra charge. See back for prizes. YES  NO

5 **Multiplier Option**  
Win up to 10X the prize! The cost will increase \$1 for every \$1 Keno wagered. YES  NO

6 **Bulls-Eye Option**  
Play for bigger prizes and more chances to win! Bulls-Eye prizes are added to Keno or Special Keno prizes. The cost will increase \$1 for every \$1 Keno wagered. YES  NO

7 Pick your own numbers or Quick Pick (QP).  
1 2 3 4 5 6 7 8 9 10  
11 12 13 14 15 16 17 18 19 20  
21 22 23 24 25 26 27 28 29 30  
31 32 33 34 35 36 37 38 39 40  
41 42 43 44 45 46 47 48 49 50  
51 52 53 54 55 56 57 58 59 60  
61 62 63 64 65 66 67 68 69 70  
71 72 73 74 75 76 77 78 79 80

8 **Play Keno To Go!**  
Pick the time of day today for games to begin!  
If no time is marked, your games begin now.  
6 7 8 9 10 11 12 1 2 3  
4 5 6 7 8 9 10 11 12 1

**KENO PRIZE PAYOUTS**  
Example Ticket Cost: Bulls-Eye can be played with Multiplier.  
Wager would be \$1 + \$1 Multiplier + \$1 Bulls-Eye = \$3 Total Cost for Wager.

MATCH	10 SPOT GAME			6 SPOT GAME		
	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**
10	\$200,000*	\$1,000,000*	\$300,000	6	\$1,600	\$2,500
9	\$4,500	\$22,000	\$15,500	5	\$95	\$95
8	\$500	\$1,200	\$1,000	4	\$5	\$2
7	\$55	\$140	\$95	3	\$1	\$0
6	\$10	\$4	\$25	2	\$0	\$0
5	\$2	\$0	\$5	1	\$0	\$0
4	\$0	\$0	\$5			
3	\$0	\$0	\$2			
2	\$0	\$0	\$2			
1	\$0	\$0	\$5			
0	\$5	\$0	\$0			
	*1-9.05	*1-75.57	**1-8.00			

MATCH	9 SPOT GAME			5 SPOT GAME		
	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**
9	\$50,000	\$75,000	\$100,000	5	\$465	\$800
8	\$3,000	\$4,000	\$5,000	4	\$15	\$12
7	\$215	\$300	\$345	3	\$2	\$0
6	\$25	\$40	\$50	2	\$0	\$0
5	\$4	\$2	\$11	1	\$0	\$0
4	\$1	\$0	\$5			
3	\$0	\$0	\$2			
2	\$0	\$0	\$2			
1	\$0	\$0	\$5			
	*1-6.53	*1-25.68	**1-8.89			

MATCH	8 SPOT GAME			4 SPOT GAME		
	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**
8	\$15,000	\$25,000	\$40,000	4	\$72	\$140
7	\$600	\$1,750	\$1,400	3	\$5	\$5
6	\$60	\$75	\$140	2	\$1	\$0
5	\$10	\$5	\$27	1	\$0	\$0
4	\$2	\$0	\$5			
3	\$0	\$0	\$2			
2	\$0	\$0	\$2			
1	\$0	\$0	\$5			
	*1-6.53	*1-25.68	**1-8.89			

MATCH	7 SPOT GAME			3 SPOT GAME		
	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**
7	\$5,500	\$7,500	\$12,000	3	\$27	\$47
6	\$150	\$330	\$400	2	\$2	\$0
5	\$15	\$20	\$60	1	\$0	\$0
4	\$2	\$1	\$10			
3	\$1	\$0	\$3			
2	\$0	\$0	\$2			
1	\$0	\$0	\$5			
	*1-4.23	*1-16.24	**1-11.43			

MATCH	6 SPOT GAME			2 SPOT GAME		
	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**
6	\$1,600	\$2,500	\$3,200	2	\$11	\$11
5	\$95	\$90	\$95	1	\$0	\$0
4	\$5	\$2	\$2			
3	\$1	\$0	\$0			
2	\$0	\$0	\$0			
1	\$0	\$0	\$4			
	*1-8.19	*1-31.48	**1-13.33			

MATCH	5 SPOT GAME			1 SPOT GAME		
	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**	KENO PRICE*	SPECIAL KENO PRICE*	BULLS-EYE PRICE**
5	\$465	\$800	\$1,035	1	\$2.50	\$2.50
4	\$15	\$12	\$10			
3	\$2	\$0	\$0			
2	\$0	\$0	\$4			
1	\$0	\$0	\$4			
	*1-10.34	*1-78.51	**1-16.00			

**ROLLING JACKPOT BONUS -**  
Match 8 of 8 and win the 8 of 8 prize PLUS the Jackpot Bonus prize for this game (shown on Keno monitors).  
\*1-9.77 \*1-48.00 \*\*1-10.00

**ROLLING JACKPOT BONUS -**  
Match 6 of 6 and win the 6 of 6 prize PLUS the Jackpot Bonus prize for this game (shown on Keno monitors).  
\*1-9.77 \*1-48.00 \*\*1-10.00

**ROLLING JACKPOT BONUS -**  
Match 4 of 4 and win the 4 of 4 prize PLUS the Jackpot Bonus prize for this game (shown on Keno monitors).  
\*1-9.77 \*1-48.00 \*\*1-10.00

**ROLLING JACKPOT BONUS -**  
Match 2 of 2 and win the 2 of 2 prize PLUS the Jackpot Bonus prize for this game (shown on Keno monitors).  
\*1-9.77 \*1-48.00 \*\*1-10.00

\*Keno Odds \*\*Special Keno Odds \*\*\*Bulls-Eye Odds  
See Keno how-to-play brochure for complete details.

**RULES:** The ticket, not the game slip, shall be the only valid proof of the selections made and the only valid receipt for claiming a prize. Must be 18 years or older to play. All claims are subject to state law and Oregon Lottery rules as authorized by ORS 662. Go to [oregonlottery.org](http://oregonlottery.org) for game rules.

oregonlottery.org ODP Rev 02/16 08-0002 03 42

Figure 1 – Oregon Lottery Keno Game Slip

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First, the player completes each section on the front of the game slip:

1. The number of spots to play, between 1 and 10.
2. The amount to wager per game: \$1, \$2, \$3, \$4, \$5, \$10, or \$20.
3. The number of consecutive games to play: 1, 2, 3, 4, 5, 10, 15, 20, 50, or 100.
4. Whether to use the special keno pay table or the standard pay table.
5. Whether or not to purchase the multiplier option.
6. Whether or not to purchase the Bulls-Eye option.
7. The numbers to play. There is also a “quick pick” option where the system randomly selects the numbers for the player.

There is also an optional starting time selection that allows the player to choose the time today when the games will begin. If no selection is made here, the player’s games begin with the next game. Oregon Lottery Keno has a maximum wager per ticket of \$100. So if you want to play 50 consecutive games, for example, you cannot wager more than \$2 per game.

The player then gives the completed game slip and the total amount wagered to a clerk at the retail establishment. The clerk inserts the game slip into a terminal that scans the game slip and prints a corresponding bar-coded ticket like the one shown in Figure 2, which is given to the player. Some locations offer player-operated vending machines that accept game slips and dispense tickets.

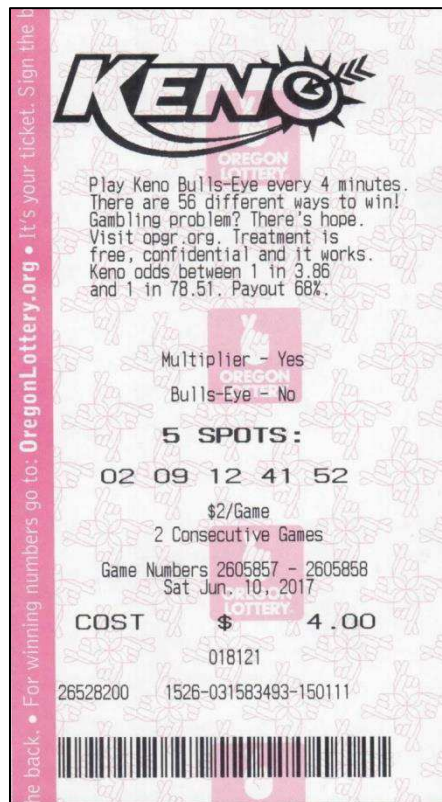


Figure 2 – Oregon Lottery Keno Ticket

### Pay Tables

Oregon Lottery Keno offers a default pay table along with second pay table called the Special Keno Prize Structure. For simplicity, these will be called the standard and special pay tables in this report. The standard pay table is a more traditional keno pay table, while the special pay

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table offers higher payouts at the expense of some lower payouts when playing 3 or more spots (the two pay tables are identical when playing 1 or 2 spots). Figure 3 shows the two pay tables for a \$1 wager.

Play 1 Spot		
Spots Hit	Pay Table	
	Standard	Special
1	\$2.50	\$2.50

Play 2 Spots		
Spots Hit	Pay Table	
	Standard	Special
2	\$11	\$11

Play 3 Spots		
Spots Hit	Pay Table	
	Standard	Special
2	\$2	-
3	\$27	\$47

Play 4 Spots		
Spots Hit	Pay Table	
	Standard	Special
2	\$1	-
3	\$5	\$5
4	\$72	\$140

Play 5 Spots		
Spots Hit	Pay Table	
	Standard	Special
3	\$2	-
4	\$15	\$12
5	\$465	\$800

Play 6 Spots		
Spots Hit	Pay Table	
	Standard	Special
3	\$1	-
4	\$5	\$2
5	\$55	\$90
6	\$1,600	\$2,500

Play 7 Spots		
Spots Hit	Pay Table	
	Standard	Special
3	\$1	-
4	\$2	\$1
5	\$15	\$20
6	\$150	\$330
7	\$5,500	\$7,500

Play 8 Spots		
Spots Hit	Pay Table	
	Standard	Special
4	\$2	-
5	\$10	\$5
6	\$60	\$75
7	\$600	\$1,750
8	\$15,000	\$25,000

Play 9 Spots		
Spots Hit	Pay Table	
	Standard	Special
4	\$1	-
5	\$4	\$2
6	\$25	\$40
7	\$215	\$300
8	\$3,000	\$4,000
9	\$50,000	\$75,000

Play 10 Spots		
Spots Hit	Pay Table	
	Standard	Special
0	\$5	-
5	\$2	-
6	\$10	\$4
7	\$55	\$140
8	\$500	\$1,200
9	\$4,500	\$22,000
10	\$200,000	\$1,000,000

**Figure 3 – Oregon Lottery Keno Standard and Special Pay Tables**

If the player wagers more than \$1 per game, the above awards are multiplied by that amount. However, the maximum award per ticket is capped at \$1,000,000 per game.

For either pay table, hitting 7 out of 7, 8 out of 8, 9 out of 9, 9 out of 10, or 10 out of 10 spots are designated as separate prize categories. If the total payout for all tickets in the same prize category exceeds \$1,000,000 in the same game, the award for that prize category is reduced to \$1,000,000 for that game, and the \$1,000,000 payout is pro-rated between the winners according to the base amount each player wagered in that game.

### Calculating the Return to Player

The return to player (RTP) is the percentage of money taken in that is paid back to the player for all possible outcomes of a game. For example, a game with an RTP of 94% means that over the long haul, one can expect that 94% of the money taken in is returned to the player, and 6% is retained. It does *not* mean that if a single player makes wagers totaling \$100 that he can expect to receive \$94 in payouts.

To calculate the RTP for a keno game, we must first calculate the probability of achieving each possible outcome for the number of spots played. For example, if we are calculating the RTP for a 4-spot keno ticket, we first calculate the probability of hitting exactly 0, 1, 2, 3, and 4 spots. This is easiest done using a branch of mathematics called *combinatorics*, which includes a way to calculate the number of ways to choose  $k$  objects from a group of  $n$  objects. This number, read aloud as “ $n$  choose  $k$ ”, is often represented by the notation:

$$C \binom{n}{k}$$

and is calculated by the formula:

$$C \binom{n}{k} = \frac{n \times (n - 1) \times (n - 2) \times \dots \times (n - k + 1)}{k \times (k - 1) \times (k - 2) \times \dots \times 1}$$

So if  $N$  is the number of spots played and  $H$  is the number of spots hit, the probability of hitting exactly  $H$  of  $N$  spots is given by this formula:

$$\frac{C \binom{20}{H} \times C \binom{60}{N - H}}{C \binom{80}{N}}$$

Returning to our 4-spot example, let’s first calculate the probability of hitting exactly 3 out of 4 spots. The above formula states that this probability is the number of ways to choose 3 numbers from the 20 called numbers, multiplied by the number of ways to choose 1 number (4 minus 3) from the remaining 60 numbers, divided by the total number of ways to choose 4 numbers from the domain of 80 numbers. This yields a probability of 0.0432479, or a frequency of 4.32479%.

Next, we need to know the RTP contribution for hitting 3 out of 4 spots. This is simply the frequency of that outcome multiplied by the associated award. In both Oregon Lottery Keno pay tables, hitting 3 out of 4 has an award of 5 for 1. So the RTP contribution for hitting 3 out of 4 spots is 4.32479% multiplied by 5, or 21.62%.

To calculate the overall RTP for a 4-spot keno ticket, we calculate the RTP contribution for each outcome – that is, hitting 0, 1, 2, 3, and 4 spots – and sum those individual contributions to get the overall RTP. Figure 4 shows the individual RTP contributions and the overall RTP for a 4-spot ticket using the standard pay table in Oregon Lottery Keno.

Play 4 Spots			
Hits	Frequency	Standard Pay Table Award RTP	
0	30.83214%	0	0.00%
1	43.27318%	0	0.00%
2	21.26355%	1	21.26%
3	4.32479%	5	21.62%
4	0.30634%	72	22.06%
Total	100.00%		64.94%

**Figure 4 – RTP Calculations for Playing 4 Spots, Standard Pay Table**

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So the Oregon Lottery Keno standard pay table has an RTP of 64.94% for the 4-spot ticket – not exactly favorable.

It should be mentioned that the frequency of hitting a given number of spots is completely independent of the pay table used, and that the frequencies for all cases must total 100%. Because we can't change the frequency of each keno outcome, we must change the award for each outcome (that is, the pay table) if we want to change the RTP.

### Calculating the Hit Frequency

The hit frequency is the percentage of games where the player wins something, even if the amount won is less than the amount wagered. Analogous to RTP, a game with a hit frequency of 25% means that over the long haul, one can expect that 25% of the games played will result in a payout. It does not mean that a single player who plays 100 games will receive a payout in 25 of those games.

To calculate the hit frequency for a keno game, we can use the frequencies calculated when determining the RTP. We simply sum the frequencies where the associated award is greater than zero. So referring back to Figure 4 for our 4-spot keno ticket using the standard pay table, the player wins something when hitting 2, 3, or 4 out of 4 spots, and the frequencies for those outcomes are 21.26355%, 4.32479%, and 0.30634% respectively. Summing those three values and rounding the result to two decimal places gives a hit frequency of 25.89% for a 4-spot keno ticket using the standard pay table.

Hit frequency can also be expressed as the odds of receiving a payout in a game, which is simply the reciprocal of the hit frequency percentage. So for the 4-spot keno ticket using the standard pay table, this is  $1 / 25.89$ , or 3.86. We can thus say that the chance of winning something is 1 in 3.86, or 1:3.86, which matches the published odds shown on the back of an Oregon Lottery Keno ticket when playing 4 spots and using the standard pay table.

### Hit Frequency and Return to Player Summary

Figure 5 shows the hit frequency and RTP values for playing 1 through 10 spots for each pay table in Oregon Lottery Keno. These values were calculated as described for the 4-spot ticket in the previous two sections.

Spots Played	Standard Pay Table			Special Pay Table		
	Hit Freq	Once Per	RTP	Hit Freq	Once Per	RTP
1	25.00%	4.00	62.50%	25.00%	4.00	62.50%
2	6.01%	16.63	66.14%	6.01%	16.63	66.14%
3	15.26%	6.55	65.21%	1.39%	72.07	65.21%
4	25.89%	3.86	64.94%	4.63%	21.59	64.51%
5	9.67%	10.34	64.91%	1.27%	78.51	66.10%
6	16.16%	6.19	64.91%	3.18%	31.48	65.81%
7	23.66%	4.23	65.30%	6.16%	16.24	64.96%
8	10.23%	9.77	64.95%	2.08%	48.00	65.85%
9	15.31%	6.53	64.87%	3.89%	25.68	65.62%
10	11.05%	9.05	65.29%	1.32%	75.57	68.08%
Average	15.82%	6.32	64.90%	5.49%	18.20	65.48%

**Figure 5 – Hit Frequency and RTP Summary**



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As shown in Figure 5, the RTP values for the standard pay table range from a low of 62.50% for a 1-spot ticket to a high of 66.14% for a 2-spot ticket, with an average RTP of 64.90% for 1 through 10 spots. The RTP values for the special pay table range from a low of 62.50% for a 1-spot ticket to a high of 68.08% for a 10-spot ticket, with a slightly higher 65.48% average RTP.

Figure 5 further reveals that the hit frequencies for the standard pay table range from a low of 6.01% for a 2-spot ticket to a high of 25.89% for a 4-spot ticket, with an average hit frequency of 15.82% for 1 through 10 spots. The hit frequencies for the special pay table range from a low of 1.27% for a 5-spot ticket to a high of 25.00% for a 1-spot ticket, with an average hit frequency of 5.49%. This is as expected -- the intent of the special pay table is to provide higher awards at the expense of some lower awards, meaning payouts are less frequent when using the special pay table.

The Oregon Lottery does not publish individual RTP values, but it does provide individual hit frequencies on the keno ticket itself and on the Oregon Lottery website. The published hit frequencies are expressed in the 1:N form, where N represents a value from the "Once Per" column in Figure 5. The published hit frequencies for playing 1 through 10 spots exactly match those in Figure 5 for both the standard and special pay tables.

### Multiplier Option

Oregon Lottery Keno includes an optional multiplier feature that multiplies the standard and special pay table awards by a factor between 1 and 10. If purchased, the multiplier option applies to all games played on that ticket and increases the total ticket cost by \$1 for each \$1 wagered in the base keno game. A new multiplier is randomly selected for each game and is revealed at the start of that game.

While the multiplier selection is random, it is weighted so lower multipliers occur more often than higher multipliers. Figure 6 shows the multiplier frequencies published on the Oregon Lottery Keno website.

Multiplier	Frequency
1x	45.0%
2x	28.3%
3x	16.0%
5x	9.7%
10x	1.0%
Total	100.0%

**Figure 6 – Published Multiplier Frequencies**

A glimpse at Figure 6 shows that the 1x multiplier occurs in 45% of the games, while a multiplier of at least 2x occurs in 55% of the games. But a more useful piece of information is determining the average multiplier for all games. To do this, we simply calculate the expected contribution of each multiplier value by multiplying it by its associated frequency, and then summing the results. This is shown in Figure 7.

Multiplier	Frequency	Contribution
1x	45.0%	0.450
2x	28.3%	0.566
3x	16.0%	0.480
5x	9.7%	0.485
10x	1.0%	0.100
Total	100.0%	2.081

**Figure 7 – Average Multiplier Value**

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Since the cost of purchasing the multiplier option doubles the bet per game, the average multiplier thus provides an expected return of 2.081 for a two-unit bet, or 1.0405 per unit bet, so purchasing the multiplier option is favorable to the player. Whether or not this was by design is not clear, but Figure 8 shows a set of comparable frequencies that could have been used to achieve an average multiplier of exactly 2 and thus provide no advantage to the player or the house.

Multiplier	Frequency	Contribution
1x	48.0%	0.480
2x	29.0%	0.580
3x	13.0%	0.390
5x	9.0%	0.450
10x	1.0%	0.100
<b>Total</b>	<b>100.0%</b>	<b>2.000</b>

**Figure 8 – Achieving an Average Multiplier of Exactly 2**

Figure 9 shows the RTP values for both pay tables when the player purchases the multiplier option (the multiplier option has no effect on the hit frequency because it does not add any awards). These are simply the RTP values in Figure 5 multiplied by the multiplier contribution of 1.0405 per unit bet. When rounded, the average RTP values for both pay tables (67.53% and 68.13%) match the published RTP (68%) for Oregon Lottery Keno. This means a player can only achieve the advertised RTP if they purchase the multiplier option.

Spots Played	Standard Pay Table			Special Pay Table		
	Hit Freq	Once Per	RTP	Hit Freq	Once Per	RTP
1	25.00%	4.00	65.03%	25.00%	4.00	65.03%
2	6.01%	16.63	68.82%	6.01%	16.63	68.82%
3	15.26%	6.55	67.86%	1.39%	72.07	67.86%
4	25.89%	3.86	67.57%	4.63%	21.59	67.12%
5	9.67%	10.34	67.54%	1.27%	78.51	68.78%
6	16.16%	6.19	67.54%	3.18%	31.48	68.48%
7	23.66%	4.23	67.94%	6.16%	16.24	67.59%
8	10.23%	9.77	67.58%	2.08%	48.00	68.51%
9	15.31%	6.53	67.50%	3.89%	25.68	68.28%
10	11.05%	9.05	67.94%	1.32%	75.57	70.84%
<b>Average</b>	<b>15.82%</b>	<b>6.32</b>	<b>67.53%</b>	<b>5.49%</b>	<b>18.20</b>	<b>68.13%</b>

**Figure 9 – RTP Summary with Multiplier Option**

Note that if Oregon Lottery Keno used the multiplier frequencies shown in Figure 8 instead of the published frequencies shown in Figure 7, the RTP values for both pay tables would match those shown in Figure 5. This means the RTP values would be the same whether or not the player purchases the multiplier option.

Multiplied awards are subject to the \$1,000,000 payout limit per ticket per game. For example, a 2x multiplier does not increase this limit to \$2,000,000 if the player purchases the multiplier option. This will be analyzed in detail later in this report.

### **Bulls-Eye Option**

Oregon Lottery Keno added a new feature called the Bulls-Eye option in mid-2017. At the conclusion of each keno game, one of the 20 called numbers is randomly selected as the “Bulls-

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Eye” number. If the player purchases the Bulls-Eye option, and the Bulls-Eye number is one of the player’s selected numbers, the player wins an award according to the pay table shown in Figure 10 (payouts are for a \$1 wager).

Play 1 Spot		Play 6 Spots		Play 9 Spots	
Hits	Payout	Hits	Payout	Hits	Payout
1	\$49.50	1	\$4	1	\$5
		2	\$3	2	\$2
		3	\$9	3	\$2
		4	\$20	4	\$5
		5	\$95	5	\$11
		6	\$3,200	6	\$50
				7	\$345
				8	\$5,000
				9	\$100,000
Play 2 Spots		Play 7 Spots		Play 10 Spots	
Hits	Payout	Hits	Payout	Hits	Payout
1	\$15	1	\$5	1	\$5
2	\$60	2	\$2	2	\$2
		3	\$3	3	\$2
		4	\$10	4	\$5
		5	\$60	5	\$5
		6	\$400	6	\$25
		7	\$12,000	7	\$95
				8	\$1,000
				9	\$15,500
				10	\$300,000
Play 3 Spots		Play 8 Spots			
Hits	Payout	Hits	Payout		
1	\$8	1	\$5		
2	\$15	2	\$2		
3	\$128	3	\$2		
		4	\$5		
		5	\$27		
		6	\$140		
		7	\$1,400		
		8	\$40,000		
Play 4 Spots					
Hits	Payout				
1	\$5				
2	\$10				
3	\$20				
4	\$318				
Play 5 Spots					
Hits	Payout				
1	\$4				
2	\$4				
3	\$10				
4	\$70				
5	\$1,035				

**Figure 10 – Oregon Lottery Keno Bulls-Eye Option Pay Table**

If purchased, the Bulls-Eye option applies to all games played on that ticket and increases the total ticket cost by \$1 for each \$1 wagered in the base keno game. The Bulls-Eye award is in addition to any other award won from the standard or special pay tables. The multiplier option does not apply to the Bulls-Eye awards.

The Bulls-Eye option provides a last chance to win something in a given game, even if the player is mathematically eliminated from winning one of the standard or special pay table awards. For example, suppose you’re playing a 10-spot ticket, 17 numbers have been called, and you’ve only caught one number. Even if you hit the remaining three numbers, you would only catch 4 out of 10 spots, which pays nothing on either pay table. But if the Bulls-Eye number matches any of your hit numbers, you win the associated Bulls-Eye award.

To calculate the RTP for the Bulls-Eye option, we’ll first determine the probability of matching the Bulls-Eye number when hitting a given number of spots. This step is surprisingly quite easy. Regardless of the number of spots played, suppose you hit one of the 20 called numbers. The Bulls-Eye number is also one of these same 20 numbers, so there is a 1 in 20 (5%) chance that the Bulls-Eye number will be the one number you hit. Similarly, if you hit two of the 20 called



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numbers, there is a 2 in 20 (10%) chance that the Bulls-Eye number will be one of the two numbers you hit. Going further, if you hit three of the called numbers, there is a 3 in 20 (15%) chance of matching the Bulls-Eye number. The pattern is apparent: the chance that one of the hit numbers matches the Bulls-Eye number is 5% times the number of hits. Or put another way, the chance of matching the Bulls-Eye increases by 5% for each number hit. This is true regardless of the number of spots played.

The second step in calculating the Bulls-Eye RTP is to multiply the probability of matching the Bulls-Eye number for a given number of spots by the probability of hitting that number of spots in the first place. Referring back to Figure 4, there is a 21.26355% chance of hitting two out of four numbers. When hitting two numbers, the probability of matching the Bulls-Eye number is 10%, so the probability of matching the Bulls-Eye number when hitting two out of four numbers is 10% times 21.26355%, or 2.12635%.

The final step is to calculate the combined probabilities as described above for each possible outcome (hitting 0 through N spots for an N-spot ticket), multiplying each combined probability by its associated award from the Bulls-Eye pay table, and summing the results. Figure 11 presents these calculations and shows the Bulls-Eye option has a hit frequency of 5.00% and an RTP of 64.54% for a 4-spot ticket.

<b>Play 4 Spots</b>					
<b>Hits</b>	<b>Frequency</b>	<b>Bulls-Eye Frequency</b>	<b>Combined Frequency</b>	<b>Bulls-Eye Option Award</b>	<b>RTP</b>
0	30.83214%	0.00000%	0.00000%	0	0.00%
1	43.27318%	5.00000%	2.16366%	5	10.82%
2	21.26355%	10.00000%	2.12635%	10	21.26%
3	4.32479%	15.00000%	0.64872%	20	12.97%
4	0.30634%	20.00000%	0.06127%	318	19.48%
<b>Total</b>	<b>100.00%</b>		<b>5.00%</b>		<b>64.54%</b>

**Figure 11 – Bulls-Eye RTP Calculations for Playing 4 Spots**

Figure 12 shows the hit frequency and RTP values for the Bulls-Eye option when playing 1 through 10 spots. These values were calculated using the procedure just described for the 4-spot ticket.

<b>Spots Played</b>	<b>Bulls-Eye Option</b>		
	<b>Hit Freq</b>	<b>Once Per</b>	<b>RTP</b>
1	1.25%	80.00	61.88%
2	2.50%	40.00	64.56%
3	3.75%	26.67	64.69%
4	5.00%	20.00	64.54%
5	6.25%	16.00	65.14%
6	7.50%	13.33	65.20%
7	8.75%	11.43	64.72%
8	10.00%	10.00	64.93%
9	11.25%	8.89	65.11%
10	12.50%	8.00	64.91%
<b>Average</b>	<b>6.88%</b>	<b>14.55</b>	<b>64.57%</b>

**Figure 12 – Bulls-Eye Hit Frequency and RTP Summary**

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As shown in Figure 12, the RTP values for the Bulls-Eye range from a low of 61.88% for a 1-spot ticket to a high of 65.20% for a 6-spot ticket, with an average RTP of 64.57% for 1 through 10 spots. Note how the Bulls-Eye hit frequencies are exactly linear, ranging from 1.25% for a 1-spot ticket to 12.50% for a 10-spot ticket, with an average hit frequency of 6.88%. The published Bulls-Eye hit frequencies for playing 1 through 10 spots exactly match those in Figure 12.

For the Bulls-Eye option, hitting 7 out of 7, 8 out of 8, 9 out of 9, 9 out of 10, or 10 out of 10 spots are designated as separate prize categories. If the total Bulls-Eye payout for all tickets in the same prize category exceeds \$1,000,000 in the same game, the Bulls-Eye award for that prize category is reduced to \$1,000,000 for that game, and the \$1,000,000 payout is pro-rated between the winners according to the base amount each player wagered in that game.

### Payout Limit and its Effect on the RTP

We've mentioned several times that Oregon Lottery Keno has a payout limit of \$1,000,000 per ticket per game. This limit effectively reduces the RTP for certain awards when combined with the amount wagered and the multiplier. To illustrate this effect with a simple example, the special pay table payout for hitting 10 out of 10 spots is \$1,000,000 for a \$1 wager with no multiplier. But a player who selects the special pay table and hits 10 out of 10 still wins \$1,000,000 even if they wager more than \$1, or if they purchase the multiplier option and the multiplier is greater than 1. The rest of this section provides an analysis of such payout constraints and their resulting effect on the RTP for both pay tables. For simplicity, we'll assume the player does not purchase the Bulls-Eye option.

Recall that Oregon Lottery Keno lets the player wager up to \$20 per game, and if they purchase the multiplier option, the maximum multiplier is 10x. So if we divide the \$1,000,000 maximum payout by the maximum bet (\$20), and then again divide by the maximum multiplier (10), we get:

$$\frac{\$1,000,000}{\$20 \times 10} = 5,000$$

This means that any payout greater than 5,000 for 1 is subject to the \$1,000,000 payout limit for large enough bets and/or large enough multipliers. Figure 13 shows all such awards for the standard and special pay tables.

Spots Played	Spots Hit	Standard Pay Table	Special Pay Table
7	7	5,500	7,500
8	8	15,000	25,000
9	9	50,000	75,000
9	10	--	22,000
10	10	200,000	1,000,000

**Figure 13 – Awards Over 5,000 for 1**

Let's calculate the payout when a player wagers \$10 on a 7-spot ticket using the standard pay table, purchases the multiplier option, and hits 7 out of 7 with a 10x multiplier. Hitting 7 out of 7 pays 5,500 for 1, so the base award (without the multiplier) is 5,500 times \$10, or \$55,000. Adding the 10x multiplier increases the final payout to \$550,000 – well under the \$1,000,000 limit.

Now let's assume the same scenario, but this time the player wagers \$20 instead of \$10. The base award then becomes \$110,000 and the final payout should be \$1,100,000. But because that amount is over the \$1,000,000 limit, the player receives \$1,000,000 and not \$1,100,000. As you might expect, that specific case rarely occurs, but it has enough impact to reduce the

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overall standard pay table RTP for a 7-spot ticket from 67.94% to 67.88% when a player wagers \$20 per game.

To calculate this RTP reduction, we first need to know the RTP contribution for hitting 7 out of 7 spots using the standard pay table. Using combinatorics as we did previously to calculate the frequency of outcomes for the 4-spot ticket, the frequency of hitting 7 out of 7 is 0.00244%. The standard pay table award for this outcome is 5,500 for 1, so the RTP contribution for hitting 7 out of 7 is the frequency (0.00244%) multiplied by the award (5,500), or 13.42%.

Next, we need to determine the average *effective* multiplier for a \$20 wager when hitting 7 out of 7 spots. The effective multiplier is ratio of the actual award to the full award, multiplied by the actual multiplier. For actual multipliers of 1x through 5x, the effective multiplier is the actual multiplier because the actual award is the full award (that is, the full award is less than the \$1,000,000 payout limit). But for a multiplier of 10x, the full award is \$1,100,000 (5,500 x \$20 x 10) but the actual award is the \$1,000,000 payout limit. So the effective multiplier in this case is 9.091 (10 x \$1,000,000 / \$1,100,000). Using the multiplier frequencies from Figure 7, we can calculate the average effective multiplier as:

$$(0.450 \times 1) + (0.283 \times 2) + (0.160 \times 3) + (0.097 \times 5) + (0.010 \times 9.091) = 2.072$$

This gives an average effective multiplier of 1.036 per unit bet (2.072 / 2) because purchasing the multiplier option doubles the cost of the wager. Figure 7 showed the average multiplier when not constrained by the \$1,000,000 payout limit is 1.0405 per unit bet. So if we apply the effective to actual average multiplier ratio to the 13.42% RTP contribution, we get:

$$13.42\% \times (1.036 / 1.0405) = 13.36\%$$

This means the standard pay table RTP contribution for hitting 7 out of 7 spots is reduced from 13.42% to 13.36% when the player wagers \$20 per game, selects the multiplier option, and receives a 10x multiplier. Because this is the only case where a 7-spot ticket payout can possibly be constrained by the \$1,000,000 payout limit, the RTP contributions for hitting less than 7 out of 7 spots remain the same. So the overall RTP of 67.94% is reduced by 0.06% (13.42% minus 13.36%), which gives an overall RTP of 67.88% for this case.

Figure 13 listed the other cases where payouts might be subject to the \$1,000,000 payout limit. Using the same calculations we just outlined for the 7-spot ticket, Figure 14 shows all specific combinations of wagers and multipliers that result in RTP reductions for the standard pay table.

Spots Played	Wager	Multiplier	RTP with Multiplier Full	RTP with Multiplier Actual
7	\$20	10x	67.94%	67.88%
8	\$10	10x	67.58%	67.47%
	\$20	5x,10x		66.84%
9	\$3	10x	67.50%	67.44%
	\$4	10x		67.41%
	\$5	5x,10x		67.21%
	\$10	3x,5x,10x		66.54%
	\$20	2x,3x,5x,10x		65.54%
10	\$1	10x	67.94%	67.44%
	\$2	3x,5x,10x		67.05%
	\$3	2x,3x,5x,10x		66.70%
	\$4	2x,3x,5x,10x		66.44%
	\$5	2x,3x,5x,10x		66.28%
	\$10	1x,2x,3x,5x,10x		65.72%
	\$20	1x,2x,3x,5x,10x		65.44%

**Figure 14 – RTP Reductions, Standard Pay Table**

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Figure 15 shows all specific combinations of wagers and multipliers that result in RTP reductions for the special pay table.

Spots Played	Wager	Multiplier	RTP with Multiplier Full	RTP with Multiplier Actual
7	\$20	10x	67.59%	67.28%
8	\$5	10x	68.51%	68.40%
	\$10	5x,10x		67.66%
	\$20	3x,5x,10x		65.63%
9	\$2	10x	68.28%	68.18%
	\$3	5x,10x		67.98%
	\$4	5x,10x		67.66%
	\$5	3x,5x,10x		67.32%
	\$10	2x,3x,5x,10x		65.84%
	\$20	1x,2x,3x,5x,10x		64.43%
	10	\$1		2x,3x,5x,10x
\$2		1x,2x,3x,5x,10x	61.97%	
\$3		1x,2x,3x,5x,10x	61.04%	
\$4		1x,2x,3x,5x,10x	60.57%	
\$5		1x,2x,3x,5x,10x	60.23%	
\$10		1x,2x,3x,5x,10x	59.06%	
\$20		1x,2x,3x,5x,10x	56.36%	

**Figure 15 – RTP Reductions, Special Pay Table**

While most RTP reduction cases for both pay tables occur when the player purchases the multiplier option, the cases listed in Figure 14 and Figure 15 with a 1x multiplier also result in an RTP reduction when the player does not purchase the multiplier option. Two further points are worth noting: there are no RTP reduction cases when the player wagers \$1 per game and does not purchase the multiplier option, and larger wagers amplify the amount of the RTP reduction.

In reality, most players probably don't consider the RTP reductions when playing Oregon Lottery Keno. If a player who wagers \$5 on a 10-spot ticket and selects the special pay table is lucky enough to hit 10 out of 10 and win \$1,000,000, the player almost certainly isn't thinking "Wow, that's great, but I could have still won \$1,000,000 if I had bet \$1 instead of \$5". Of course, the appeal of larger wagers and higher multipliers is that they result in greater payouts in the vast majority of cases when playing 7 spots or more, and in all cases when playing 6 spots or less.

### Progressive Jackpot

Oregon Lottery Keno includes a progressive jackpot called the Rolling Jackpot Bonus, or simply the Jackpot Bonus. The progressive jackpot only applies to 8-spot keno tickets and is won by a player who hits all 8 of 8 spots. If a player wins the progressive jackpot, they also win the standard or special pay table award for hitting 8 out of 8, and if the player purchased the multiplier option, the multiplier applies to that award, but not to the progressive jackpot amount. If more than one player wins the Jackpot Bonus in the same game, the progressive amount is pro-rated between the winners according to the base amount each player wagered in that game. According to the Oregon Lottery Keno website, the average Jackpot Bonus award is \$30,000.

The progressive jackpot starts at zero and is fed from a fixed percentage of all standard and special pay table wagers, but not from wagers on the multiplier and Bulls-Eye options. Because it is only possible to win the jackpot on an 8-spot ticket, players who do not play 8-spot tickets are contributing to a progressive jackpot that they cannot win.

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The progressive contribution is not published. However, the Oregon Lottery Keno website states that the keno payout (that is, the RTP) is 68% including the Jackpot Bonus, so we can estimate that the progressive contribution. Figure 5 shows that the RTP for an 8-spot ticket is 64.95% when using the standard pay table and 65.85% when using the special pay table if the player does not select the multiplier option. For the 8-spot ticket to match the published 68% RTP, we must add a progressive contribution. A contribution of 2.55% for the standard pay table and 1.65% for the special pay table would bring the 8-spot RTP to 67.50% for both tables, which when rounded matches the published 68% RTP. It may be that tickets played for the two pay tables provide different progressive contributions.

The above figures don't account for the fact that the 8-spot RTP (and the average RTP) is higher when the player purchases the multiplier option. Again, this is because the average multiplier per unit bet is greater than 1 (see Figure 7 and the surrounding discussion). The data in Figure 9 show that the average RTP is already 68% if the player purchases the multiplier option, even without any progressive contribution.

### Player Simulation Metrics

To quantify the Oregon Lottery Keno player experience, computer simulations of 100,000 players were performed when playing 1 through 10 spots for each pay table / multiplier combination. In the simulations, each player wagered \$1 per game (\$2 if purchasing the multiplier option) to avoid the effects of the payout limit to the greatest extent possible. Further, players never purchased the Bulls-Eye option. Each player was given a \$20 starting bankroll (\$40 if purchasing the multiplier) and continued playing until they played 600 games or until their bankroll was exhausted, whichever occurred first. Figure 16 shows the average number of games played in each simulation.

Spots Played	Standard Pay Table		Special Pay Table	
	Normal	Multiplier	Normal	Multiplier
1	49.83	54.63	--	--
2	56.16	60.97	--	--
3	53.98	57.28	51.18	51.49
4	53.07	54.84	46.05	45.65
5	39.59	39.67	30.25	30.34
6	36.17	37.78	30.08	30.87
7	39.63	40.90	32.90	32.89
8	40.36	41.50	28.25	28.75
9	38.56	39.16	34.18	33.81
10	44.05	45.94	27.65	27.98

**Figure 16 – Average Games Played Simulation Metrics**

The simulation results reveal that the average number of games played is always greater for the standard pay table than for the special pay table (the two pay tables have identical awards for playing 1 or 2 spots). This makes perfect sense, as the average hit frequency for the standard pay table is about three times higher than the average hit frequency for the special pay table.

The simulation results in Figure 16 further reveal that purchasing the multiplier option increases the average number of games played for the standard pay table. This is as expected, since the average multiplier is 2.081 for a \$2 wager (see Figure 7). The same is generally true for the special pay table, but its much lower hit frequency makes the difference less (and in some cases the simulations actually showed a slight decrease in the average number of games played when the multiplier was purchased).



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Since each of the 100,000 simulated players had a \$20 starting bankroll and wagered \$1 per game (a \$40 bankroll wagering \$2 per game for simulations where the multiplier was purchased), each player was guaranteed to play at least 20 games. Figure 17 shows the percentage of the players remaining at six different levels – from the 20-game minimum to the 600-game maximum – for each different simulation case. For example, when playing 1 spot without purchasing the multiplier, 99.54% of the players were able to play more than 20 games (or put another way, 0.46% of such players won nothing and exhausted their bankroll in 20 games). Because a new keno game is played every four minutes, this means virtually all players who play a 1-spot ticket without purchasing the multiplier would have at least some of their original bankroll remaining after 80 minutes.

As one would expect, there is a correlation between the RTP, hit frequency, and time on device data. Looking at a simple case using the standard pay table, the 2-spot ticket has the lowest hit frequency, while the 4-spot ticket has the highest (see Figure 5). The player simulation data shows the 2-spot ticket has the lowest percentage of players to play more than 20 games (69.30%, 69.40% with multiplier) and the 4-spot ticket has the highest (98.30%, 97.88% with multiplier).

Spots Played	Purchased Multiplier?	Pay Table	Games Played					
			> 20	> 30	> 50	> 100	> 200	600
1	No	Either	99.54%	87.16%	41.13%	2.62%	0.01%	0.00%
	Yes		98.67%	81.00%	42.12%	7.62%	0.35%	0.00%
2	No	Either	69.30%	50.15%	38.03%	13.11%	3.14%	0.05%
	Yes		69.40%	48.86%	32.94%	14.95%	5.42%	0.36%
3	No	Standard	95.70%	43.48%	30.09%	12.46%	3.75%	0.17%
		Special	23.09%	23.09%	23.09%	12.66%	5.85%	0.90%
	Yes	Standard	90.54%	44.52%	24.67%	12.19%	4.98%	0.64%
		Special	23.31%	23.31%	16.65%	10.08%	5.69%	1.53%
4	No	Standard	98.30%	54.86%	15.72%	9.65%	3.81%	0.51%
		Special	59.42%	17.98%	7.55%	7.23%	5.28%	1.23%
	Yes	Standard	97.88%	52.87%	19.22%	8.36%	4.44%	0.98%
		Special	59.32%	17.79%	7.34%	7.03%	5.06%	1.20%
5	No	Standard	85.73%	27.70%	9.15%	2.29%	1.83%	1.82%
		Special	21.71%	21.71%	2.51%	1.40%	1.39%	1.39%
	Yes	Standard	75.19%	26.60%	10.30%	3.25%	2.04%	1.32%
		Special	21.41%	13.87%	3.37%	1.60%	1.48%	1.01%
6	No	Standard	87.84%	25.69%	8.53%	5.11%	1.04%	0.41%
		Special	45.97%	6.18%	6.17%	6.17%	1.93%	0.36%
	Yes	Standard	87.46%	26.92%	10.00%	4.44%	1.82%	0.52%
		Special	32.65%	6.77%	6.42%	4.11%	1.91%	0.53%
7	No	Standard	97.35%	32.94%	11.00%	2.81%	2.38%	0.29%
		Special	40.23%	17.15%	5.33%	2.08%	1.80%	0.56%
	Yes	Standard	96.75%	37.68%	12.29%	3.95%	1.84%	0.48%
		Special	45.09%	14.59%	7.08%	2.69%	1.86%	0.78%
8	No	Standard	87.23%	37.88%	12.25%	6.29%	1.76%	0.65%
		Special	32.69%	6.49%	5.11%	3.43%	0.70%	0.43%
	Yes	Standard	78.24%	32.57%	13.99%	5.43%	2.25%	0.63%
		Special	33.27%	10.11%	5.46%	2.79%	1.21%	0.46%
9	No	Standard	86.35%	25.84%	13.47%	3.09%	1.94%	0.48%
		Special	52.96%	12.15%	12.13%	4.29%	1.75%	0.55%
	Yes	Standard	86.58%	28.89%	11.20%	4.30%	1.88%	0.62%
		Special	53.06%	12.19%	12.16%	4.23%	1.72%	0.55%
10	No	Standard	89.18%	50.14%	17.49%	5.89%	1.61%	0.63%
		Special	22.39%	3.67%	3.34%	3.34%	0.98%	0.41%
	Yes	Standard	84.59%	46.47%	19.59%	6.23%	2.25%	0.56%
		Special	22.45%	4.43%	3.50%	2.44%	1.48%	0.52%

**Figure 17 – Time on Device Simulation Metrics**

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The simulation results in Figure 17 also show a substantial loss of players between 20 and 30 games. This loss is sometimes less pronounced for the special pay table because a larger player loss typically occurs before 20 games due to the much lower hit frequency of the special pay table.

### Game Strategy

The best strategy for Oregon Lottery Keno is not to play. But kidding aside, here are some basic guidelines to help you enjoy the game.

- Don't play a 1-spot ticket. It has the lowest RTP at 62.50%.
- If your primary objective is to increase your RTP, use the special pay table.
- If your primary objective is to increase the length of time you play on a fixed bankroll, use the standard pay table.
- Always purchase the multiplier option.
- Never purchase the Bulls-Eye option.
- If you're concerned about the RTP reductions described in this report, ensure the maximum payout and maximum multiplier (if purchased) aren't constrained by the \$1,000,000 payout limit. To do this, multiply the published payout by your bet amount, and if you purchase the multiplier option, multiply that result by 10 (the maximum multiplier). If the total is over \$1,000,000, bet less or don't purchase the multiplier option.

### Conclusions

Ignoring the effect of the payout limit on the RTP, our calculations showed an average RTP of 65% if the player does not purchase the multiplier option, and 68% if they do. This latter figure matches the RTP published on the Oregon Lottery Keno website. These RTP values are well below that of typical video keno games, which commonly have RTP values between 85% and 95%. We can attribute some of this difference to the fact that Oregon Lottery retailers receive a 5% to 10% commission on total sales, a 1% commission on total payouts, and an additional 1% commission on all payouts of \$10,000 or more.

An article published in *The Oregonian* (Portland, Oregon) in early 2017 and carried in some other regional newspapers discussed the premise that the demographic for lottery players was decreasing, while competition from area tribal casinos was increasing. The net effect is reduced revenue from lottery games in Oregon, presumably including Oregon Lottery Keno. The article also mentions that one state representative asked the Oregon lottery director "to look into the effects of decreasing the amount the lottery pays to players". Here's an alternative idea: rather than decreasing the RTP, increase it so the game will appeal to more players. And with more players you'll get more revenue.

### About the Author

Ted Gruber is president and co-founder of Ted Gruber Software, Inc. (TGS), a Nevada corporation specializing in the design and development of mathematical models for the gaming industry. TGS specializes in math for slot machines, bingo, keno, video poker, scratch cards, pull tabs, table games, side bets, skill games, gaming promotions, new games – you name it.

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