A Mathematical Analysis of Oregon Lottery Lucky Lines

Introduction

This report provides a detailed mathematical analysis of the Lucky Lines[SM] game offered through the Oregon Lottery (https://www.oregonlottery.org/games/draw-games/lucky-lines). Lucky Lines is a daily draw game available at restaurants, bars, and other locations throughout the state of Oregon (United States) where Oregon Lottery games are offered. The game features a pari-mutuel progressive jackpot that starts at $10,000 and increases by $1,000 each day the jackpot is not won. It costs $2 to play one game, and up to 14 consecutive games (two weeks) may be played on the same game slip.

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Game Overview

The Lucky Lines game board consists of nine squares arranged in a 3x3 grid, just like a tic tac toe game. The center square is a "free" square that provides an automatic match, and the other eight squares each contain four unique numbers. The player selects one of the four numbers from each of those eight squares. In the game draw, one of the four numbers is called from each of the eight squares. If the player's selected number matches the drawn number in a given square, the player matches that square. If the player's matched squares form a horizontal, vertical, or diagonal line, the player wins. The more lines matched, the higher the award.

A blank Lucky Lines game slip is shown in Figure 1. The player makes his or her selections on the front of the game slip. The back of the game slip shows the pay table, brief game instructions, and other relevant information. A separate brochure describes the game and explains how to play in more detail.

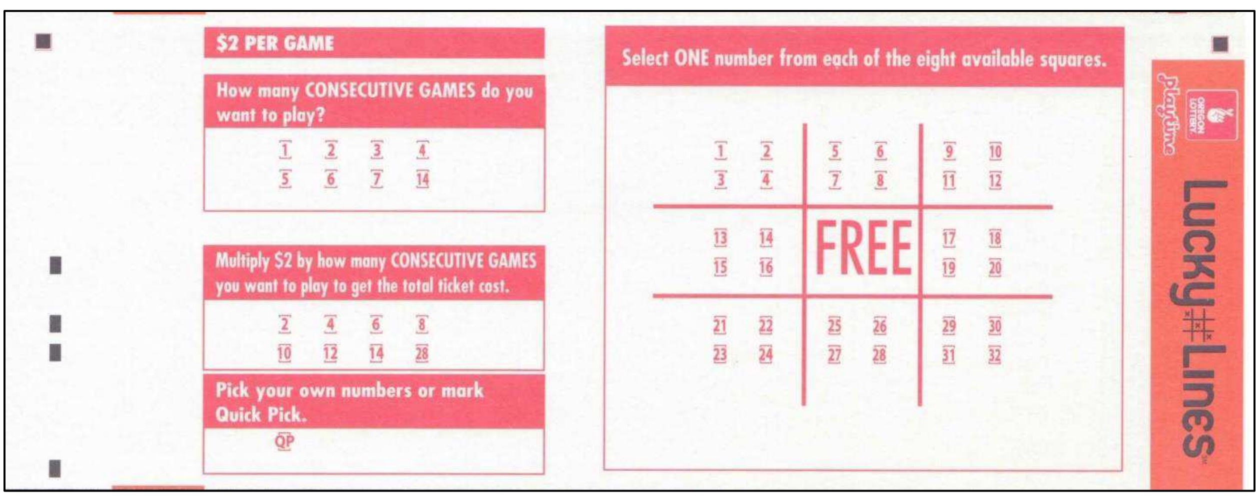

Figure 1 – Lucky Lines Game Slip

First, the player completes each section on the front of the game slip:

1. The number of consecutive games (days) to play, between 1 and 7, or 14.
2. The ticket cost ($2 times the number of games to play).
3. The selected number in each of the eight squares. There is also a “quick pick” option where the system randomly selects the numbers for the player.

The player then gives the completed game slip and the total amount wagered to a clerk at the retail establishment. The clerk inserts the game slip into a terminal that scans the game slip and prints a corresponding bar-coded ticket like the one shown in Figure 2, which is given to the player. Some locations offer player-operated vending machines that accept game slips and dispense tickets.

Figure 2 – Lucky Lines Ticket

Pay Table

The Lucky Lines pay table is shown in Figure 3. Even though the published pay tables show an award for matching seven lines, it is impossible to do so. The award for matching all eight lines is the progressive jackpot; the $10,000 award shown in the pay table is the progressive's reset (minimum) amount.

Pay Table		
Lines Matched	Award	
	$2 Bet	Per Credit
8	$10,000	5000
7	$10,000	5000
6	$500	250
5	$100	50
4	$25	12.5
3	$7	3.5
2	$4	2
1	$2	1
0	$0	0

Figure 3 – Pay Table

The ticket cost is fixed at $2 per game, but a player can of course purchase additional identical tickets to effectively wager any multiple of $2 per game.

Calculating the Return to Player

The return to player (RTP) is the percentage of money taken in that is paid back to the player for all possible outcomes of a game. For example, a game with an RTP of 94% means that over the long haul, one can expect that 94% of the money taken in is returned to the player, and 6% is retained. It does *not* mean that if a single player makes wagers totaling $100 that he can expect to receive $94 in payouts.

Before proceeding, let's assign numbers to the squares and lines in the 3x3 game grid as shown in Figure 4:

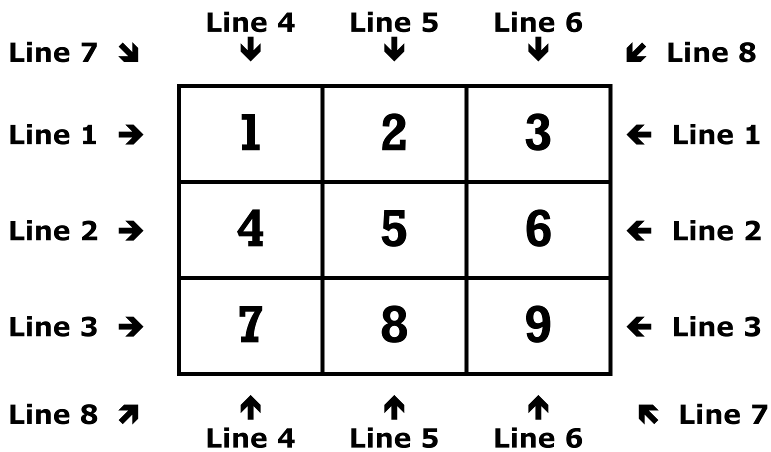

Figure 4 – Square and Line Numbering

The number of lines matched in a game depends on which individual squares were matched in that game. For each square, there are two possible outcomes: the player matched the square, or they did not. Since there are four numbers in each square and the player selects one of those numbers, the probability of the player matching a given square is 1 in 4, or 0.25. Conversely, the probability of the player not matching a given square is 3 in 4, or 0.75. Of course, the player always matches center "free" square (square 5).

Since there are eight squares in play and each square has two possible outcomes (matched or not matched), there are 2^8 or 256 different outcomes. The easiest way to calculate the Lucky Lines RTP is to calculate the RTP contribution for each of those 256 outcomes and then sum the results. Using this approach, things are simplified if we normalize the pay table awards to reflect the amount won per credit bet. For example, matching four lines pays $25 for a $2 wager, or 12.5 credits per credit bet. The pay table previously presented in Figure 3 shows both the amount won for a $2 wager and the amount won per credit.

To calculate the RTP contribution for an individual outcome, we first determine the probability of that outcome, determine how many matched lines that outcome produces, and multiply the outcome probability by the pay table award for that many lines. Let's illustrate this process with an example.

Suppose the player matches only squares 1, 2, 3, 5, and 8 in a given game, which means the player does not match squares 4, 6, 7, and 9 (this is one of the 256 possible outcomes). The probability of matching squares 1, 2, 3, and 8 is 0.25 each; the probability of NOT matching squares 4, 6, 7, and 9 is 0.75 each; and the probability of matching square 5 (the free square) is 1. So the probability of matching this particular combination of squares is:

$$0.25 \times 0.25 \times 0.25 \times 0.75 \times 1.00 \times 0.75 \times 0.75 \times 0.25 \times 0.75 = 0.001236$$

Next, we need to determine the number of lines matched for this outcome. Figure 5, which shows the squares needed to match each of the eight lines, will help with this.

Squares to Lines Relationships	
Line	**Squares Needed to Match**
1	1, 2, 3
2	4, 5, 6
3	7, 8, 9
4	1, 4, 7
5	2, 5, 8
6	3, 6, 9
7	1, 5, 9
8	3, 5, 7

Figure 5 – Squares to Lines Relationships

Using Figure 5, we see that matching squares 1, 2, 3, 5, and 8 results in matching two lines (1-2-3 and 2-5-8), giving a $4 award for a $2 wager, or two credits per credit bet. The RTP contribution for this particular outcome is thus:

$$0.001236 \times 2 = 0.002472$$

which is 0.2472%.

If we calculate the RTP contribution for the other 255 possible outcomes using this same process, and then add the results grouped by the number of lines matched, we obtain the RTP data shown in Figure 6.

Return to Player (Without Rolling Jackpot)				
Lines Matched	**Award per Credit**	**Frequency**	**Once Per**	**RTP**
8	5000	0.0015%	65,536.00	7.63%
7	5000	0.0000%	--	0.00%
6	250	0.0183%	5,461.33	4.58%
5	50	0.0458%	2,184.53	2.29%
4	12.5	0.1648%	606.81	2.06%
3	3.5	0.8514%	117.45	2.98%
2	2	3.5431%	28.22	7.09%
1	1	20.5994%	4.85	20.60%
0	0	74.7757%	1.34	0.00%
Total		100.0000%		47.22%

Figure 6 – Return to Player (Without Rolling Jackpot)

The RTP values in Figure 6 assume the reset value ($10,000, or 5000 credits per credit bet) for the Rolling Jackpot. We'll add the Rolling Jackpot contribution in the next section.

Note that none of the possible 256 outcomes result in matching exactly seven lines (indicated by a 0% frequency in Figure 6). That makes perfect sense, since there is no configuration of matching squares that results in matching exactly seven lines. Promotional materials published by the Oregon Lottery provide the same payout information for matching seven or eight lines. Most likely, lottery officials deemed this an easier solution than trying to explain why it is not possible to match exactly seven lines.

Rolling Jackpot

Lucky Lines includes a progressive jackpot called the *Rolling Jackpot* that is won by a player who matches all eight lines in one game (as noted earlier, promotional materials published by the Oregon Lottery state that a player wins the Rolling Jackpot by matching seven or eight lines, but it is impossible to match exactly seven lines). The Rolling Jackpot starts at $10,000 and increases by $1,000 per day until somebody wins the jackpot, at which time it resets to $10,000 for the next day's drawing. This behavior is quite different from traditional progressive jackpots, which increase by a fixed percentage of the total amount wagered by all players.

The Oregon Lottery website includes historical results for Lucky Lines and other games. These results include the daily amount of the Rolling Jackpot since the game began in 2008. From this data, we can calculate that between 2008 and September 30, 2017, the average jackpot won was $37,832, or 18,916 credits per credit bet. If we replace the jackpot reset amount with the average jackpot amount in Figure 6, we get the RTP data shown in Figure 7.

Return to Player (With Rolling Jackpot)				
Lines Matched	**Award per Credit**	**Frequency**	**Once Per**	**RTP**
8	18916	0.0015%	65,536.00	28.86%
7	18916	0.0000%	--	0.00%
6	250	0.0183%	5,461.33	4.58%
5	50	0.0458%	2,184.53	2.29%
4	12.5	0.1648%	606.81	2.06%
3	3.5	0.8514%	117.45	2.98%
2	2	3.5431%	28.22	7.09%
1	1	20.5994%	4.85	20.60%
0	0	74.7757%	1.34	0.00%
Total		100.0000%		68.46%

Figure 7 – Return to Player (With Rolling Jackpot)

Note that the RTP contribution for matching eight lines increases substantially when the Rolling Jackpot is considered, but the RTP contributions for matching zero through six lines remain the same as before. Further, note that the *chance* of matching eight lines did not change when we added the Rolling Jackpot. It is still once per 65,536 games, but the RTP contribution increased because the award increased.

The 68.46% calculated RTP differs from the 60.95% RTP published by the Oregon Lottery. If we assume a Rolling Jackpot award of $28,000 (14,000 credits per credit bet) instead of the average jackpot amount, we get an RTP contribution of 21.36% for eight lines matched. Using that value with the calculated RTP contributions for matching zero through six lines (39.59%) gives an RTP of 60.95%, which exactly matches the published value.

So why did Oregon Lottery officials use a Rolling Jackpot award of $28,000 instead of the average award when calculating the published Lucky Lines RTP? That isn't exactly clear, but one possibility is that they used the fact that the historical results reveal the Rolling Jackpot is won about every 28 days on average. 28 days times the $1,000 daily increase gives $28,000, but that doesn't consider the $10,000 reset amount. A more likely scenario is that lottery officials estimated how many Lucky Lines tickets would be purchased each day, and knowing that the Rolling Jackpot hits once per 65,536 games on average, calculated that somebody should win the Rolling Jackpot every 19 days. The average jackpot would then be the $10,000 reset amount on the first day, plus 18 daily $1,000 increases, for a $28,000 average jackpot. If this was the case, officials overestimated the average number of tickets sold per day.

Calculating the Hit Frequency

The hit frequency is the percentage of games where the player wins something, even if the amount won is less than the amount wagered. Analogous to RTP, a game with a hit frequency of 25% means that over the long haul, one can expect that 25% of the games played will result in a payout. It does not mean that a single player who plays 100 games will receive a payout in 25 of those games.

To calculate the Lucky Lines hit frequency, we can use the frequencies calculated when determining the RTP. We simply sum the frequencies where the associated award is greater than zero. So referring back to Figure 6, the player wins something when matching one or more lines. Summing the frequencies for those outcomes and rounding the result to two decimal places gives a hit frequency of 25.22%.

Hit frequency can also be expressed as the odds of receiving a payout in a game, which is simply the reciprocal of the hit frequency percentage. For Lucky Lines, this is 1 / 25.22, or 3.96. We can thus say that the chance of winning something is 1 in 3.96, or 1:3.96. This matches the published Lucky Lines hit frequency.

Player Simulation Metrics

To quantify the Lucky Lines player experience, computer simulations of 100,000 players were performed. Each player wagered $2 per game, was given a $40 starting bankroll, and played 600 games or until their bankroll was exhausted, whichever occurred first. Figure 8 shows some of the overall simulation results.

Average games played	31.45
Hit frequency	25.21%
Players who hit a Rolling Jackpot	0.05%

Figure 8 – Overall Simulation Results

The average number of games played (31.45) is substantially lower than the simulation metrics for most slot machines we design (typically between 120 and 160 average games played for the same simulation conditions). This can be explained by the low overall RTP (68.46%) and the fact that the largest single component (28.86%) of the RTP is for the rarely occurring Rolling Jackpot. Taking this further, players who do not hit the Rolling Jackpot have an expected RTP of less than 40%. That applies to virtually all players; the simulation revealed that 99.95% of the players did not hit the Rolling Jackpot.

Since each of the 100,000 simulated players had a $40 starting bankroll and wagered $2 per game, each player was guaranteed to play at least 20 games. Figure 9 shows the player time on device simulation results representing the percentage of players remaining at six different levels – from the 20-game minimum to the 600-game maximum. In our slot machine math models, our player simulations assume players play ten games per minute and we calculate time on device metrics at 2, 3, 5, 10, 20, and 60 minute thresholds. Using 20, 30, 50, 100, 200, and 600 games in the Lucky Lines player simulations allows us to directly compare the Lucky Lines time on device metrics to those of a typical slot machine.

Figure 9 shows that about 98% of the players were able to play more than 20 games. Or put another way, about 2% of the players won nothing and exhausted their bankroll within 20 games. Figure 9 also reveals a sharp drop in the percentage of players remaining between 20 and 30 games, and an even larger reduction between 30 and 50 games. This translates to over 97% of the players exhausting their starting bankroll before playing 50 games. If this were a slot machine, over 97% of the players would be broke within five minutes. That's not good.

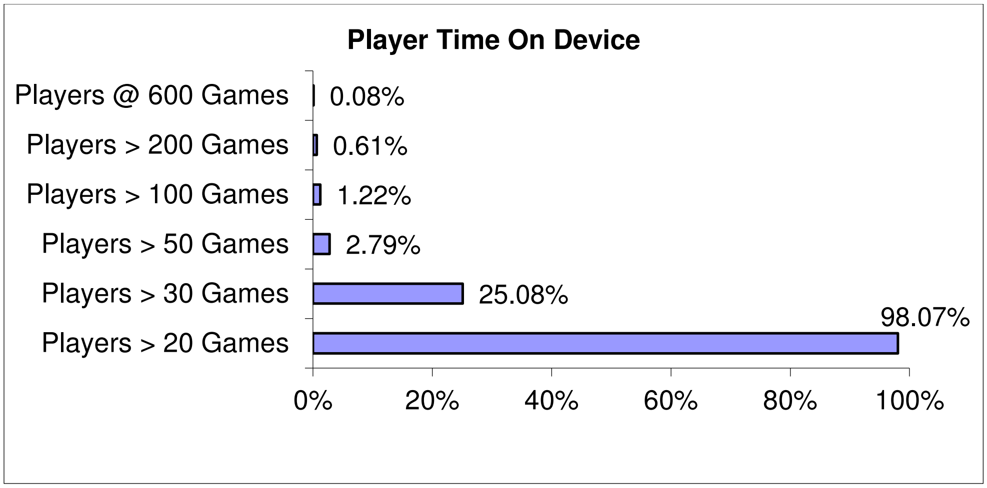

Figure 9 – Time on Device Simulation Results

Figure 10 shows the percentage of simulated players who won an award at or above selected thresholds (the chosen thresholds correspond to the awards for matching two or more lines). Note that the percentages include awards *at or above* each threshold, so if we want to know what percentage of players matched a specific number of lines as their top award, we must subtract the percentage of players for the next threshold. For example, 1.79% of the players won $100 by matching five lines, but some of those players matched six lines or hit the Rolling Jackpot. Subtracting the 0.60% of players whose top award was matching six or more lines, we find that 1.19% of simulated players had a top award of matching exactly five lines.

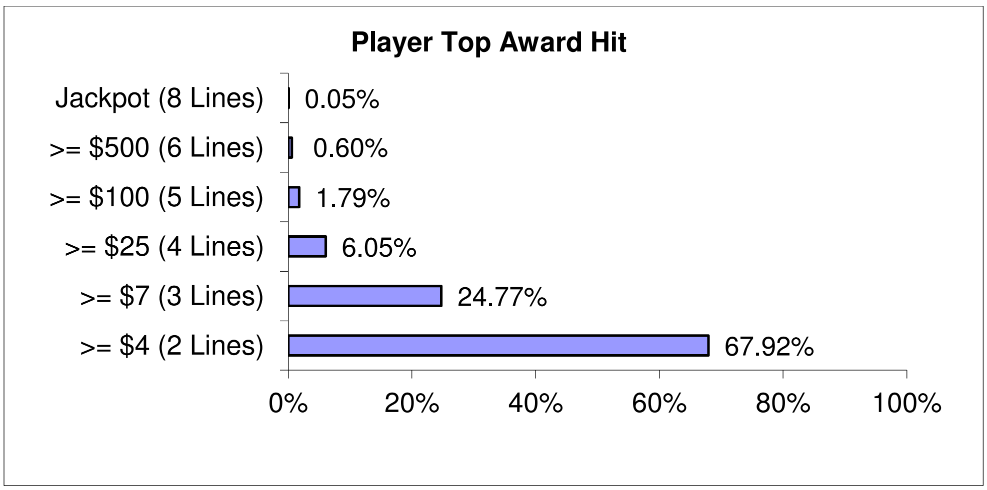

Figure 10 – Top Award Hit Simulation Results

Advantage Play Analysis

Advantage play (AP) is defined as a game state where the expected value (EV) of a given bet exceeds 100%. In other words, to determine if Lucky Lines provides AP opportunities, we need to determine if there is a pay table that provides an RTP above 100%. The awards for matching one through six lines are always the same, and collectively they provide an RTP contribution of 39.59% (see Figure 6). So, we need to find the conditions when the Rolling Jackpot provides an RTP contribution of 100% minus 39.59%, or 60.41%.

Recall that the Rolling Jackpot starts at $10,000 and increases by $1,000 each day it is not won. In theory, the Rolling Jackpot could increase infinitely, but of course it will be won at some point.

The chance of hitting the Rolling Jackpot is 65,536 to 1 (an approximate frequency of 0.0015%), so we need to find the per credit award N that satisfies the equation:

$$\frac{N}{65{,}536} = 60.41\%$$

Using elementary algebra we calculate N = 65,536 x 60.41%, or 39,590. That number is the required award per credit, so the Rolling Jackpot amount that satisfies the above equation for a $2 wager is twice that amount, or $79,180. Because the Rolling Jackpot always increases in $1,000 increments, an AP condition exists once the Rolling Jackpot reaches $80,000. Figure 11 shows the expected value for a few Rolling Jackpot amounts at and above this amount.

Jackpot	EV
$80,000	100.63%
$85,000	104.44%
$90,000	108.26%
$91,000	109.02%
$100,000	115.89%

Figure 11 – Example Expected Values for Large Rolling Jackpots

Since the Lucky Lines game began in 2008, the largest Rolling Jackpot won is $91,000. Further, the Rolling Jackpot has reached $80,000 only twice, and it has been at or above that amount for a total of just 17 days since the game's inception. When the Rolling Jackpot reaches $80,000, a player could buy all 65,536 possible Lucky Lines tickets and *almost* be guaranteed to make a profit that day. We say almost because the Rolling Jackpot is pari-mutuel, so if two or more players hit the jackpot on the same day, they share it.

Because the Rolling Jackpot rarely reaches the $80,000 threshold needed for AP conditions to exist, and the fact that the jackpot is pari-mutuel and the player bears the risk of possibly sharing the jackpot with another winner, it is safe to say that Lucky Lines is not an appealing target for advantage players.

Conclusions

The calculations presented in this report show that Lucky Lines has a true RTP of 68.46%, while the RTP published by the Oregon Lottery is 60.95%. This difference is completely attributable to our use of the average Rolling Jackpot amount instead of the fixed $28,000 amount the Oregon Lottery used in their RTP calculations. In either case, the award distribution is skewed toward high payouts because the Rolling Jackpot contributes more than one-third of the overall RTP.

Lucky Lines is vulnerable to AP conditions, but since those conditions occur so rarely and are not without substantial risk, the game really does not lend itself toward AP.

About the Author

Ted Gruber is president and co-founder of Ted Gruber Software, Inc. (TGS), a Nevada corporation specializing in the design and development of mathematical models for the gaming industry. TGS specializes in math for slot machines, bingo, keno, video poker, scratch cards, pull tabs, table games, side bets, skill games, gaming promotions, new games – you name it.

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