A Mathematical Analysis of Oregon Lottery Win for Life

Introduction

This report provides a detailed mathematical analysis of the Win for Life[SM] draw game offered through the Oregon Lottery (https://www.oregonlottery.org/games/draw-games/win-for-life). Win for Life is a three times per week draw game available at restaurants, bars, convenience stores, and other locations where lottery games are offered throughout the state of Oregon (United States). The game features a top prize of $1,000 a week as long as the winner remains alive, with a five year minimum guarantee. There are also $50,000, $20,000, $10,000, and lower prizes. The cost to play is $2 per game.

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Game Overview

To play Win for Life, a player selects a set of four different numbers between 1 and 77 for a chance to win the top prize (we'll call these four numbers the Win for Life set). The player also receives 14 other randomly-generated sets of four numbers for a chance to win lesser prizes in the same drawing. Four of the 77 numbers are called at random in the drawing, and the player wins if at least two of the called numbers match the player's numbers in the same set, or if there are no numbers matched in all 15 sets. It costs $2 to play one game for one drawing, and up to five games for up to ten consecutive drawings may be played on the same game slip.

The player wins the top prize of $1,000 a week for life ($52,000 per year) if the four player-selected numbers in the Win for Life set match the four called numbers. The top prize has a minimum duration of five years ($260,000) and is aggregated if more than three players win the top prize in the same drawing.

A blank Win for Life game slip is shown in Figure 1. The player makes his or her selections on the front side, while the back side explains how to play Win for Life, how to complete the game slip, and other pertinent information.

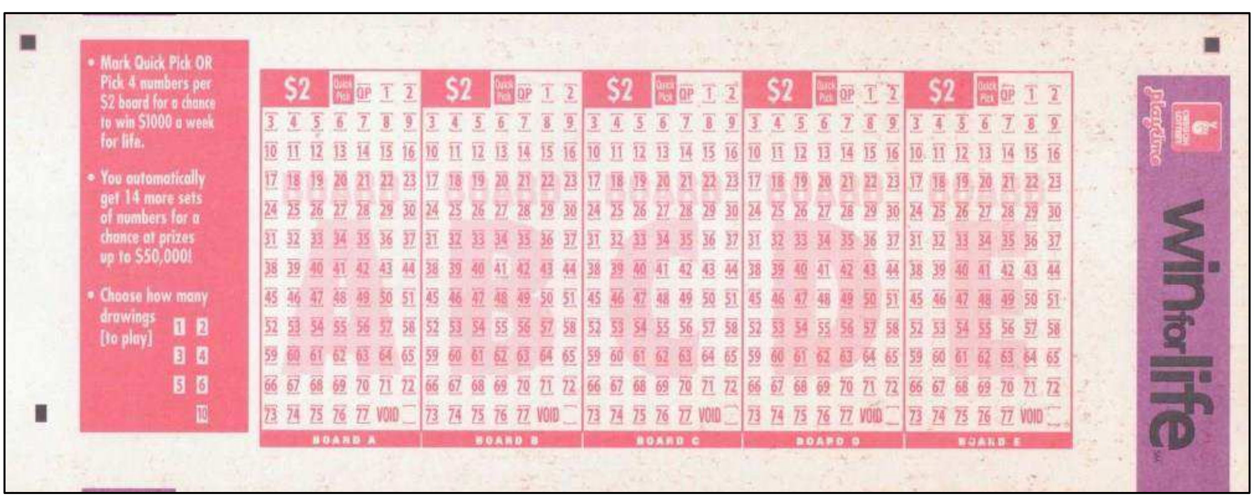

Figure 1 – Win for Life Game Slip

First, the player completes each section on the front of the game slip:

1. The number of drawings to play, between 1 and 6, or 10.

2. The numbers to play on one or more of the five game boards. The boards are labeled A through E and are provided for playing up to five games for the same drawing(s). There is also a “quick pick” option where the system randomly selects the player numbers for the Win for Life set (the player may mix or match selecting numbers or using the quick pick option on the five game boards).

The player then gives the completed game slip and the total amount wagered ($2 per filled out game board multiplied by the number of drawings to play) to a clerk at the retail establishment. The clerk inserts the game slip into a terminal that scans the game slip and prints a corresponding bar-coded ticket like the one shown in Figure 2, which is given to the player. Some locations offer player-operated vending machines that accept game slips and dispense tickets.

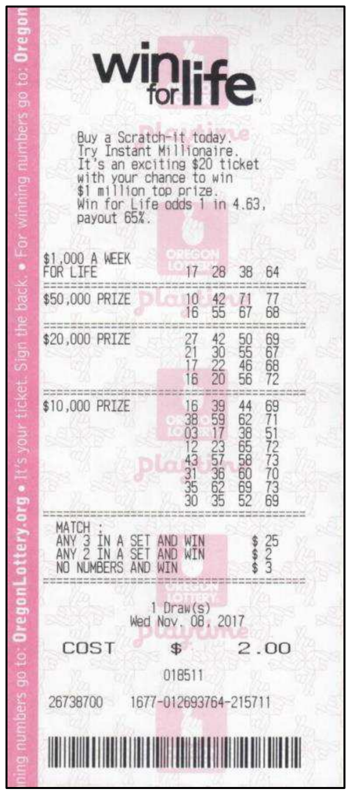

Figure 2 – Win for Life Game Ticket

Pay Table

The Win for Life pay table is shown in Figure 3.

Match Criteria	Prize ($2 Wager)
All 4 numbers in the WFL set	$1,000/week for life
All 4 numbers in a $50,000 set	$50,000
All 4 numbers in a $20,000 set	$20,000
All 4 numbers in a $10,000 set	$10,000
Any 3 numbers in a set	$25
Any 2 numbers in a set	$2
No matches in all 15 sets	$3

Figure 3 – Pay Table

The ticket cost is fixed at $2 per game, and as mentioned earlier, the player can purchase up to five entries per drawing on the same game slip. If more than three players win the top prize in the same drawing, the prize is aggregated between those players as if there were three such winners. But as we'll see, the chance of that happening is so small that we can ignore it.

Calculating the Return to Player

The return to player (RTP) is the percentage of money taken in that is paid back to the player for all possible outcomes of a game. For example, a game with an RTP of 94% means that over the long haul, one can expect that 94% of the money taken in is returned to the player, and 6% is retained. It does *not* mean that if a single player makes wagers totaling $100 that he can expect to receive $94 in payouts.

To calculate the Win for Life RTP, we'll first calculate the probability of matching exactly 0 of 4, 2 of 4, 3 of 4, and 4 of 4 numbers from a domain of 77 numbers. This is easiest done using a branch of mathematics called *combinatorics*, which includes a way to calculate the number of ways to choose k objects from a group of n objects. This number, read aloud as "n choose k", is often represented by the notation:

$$C\binom{n}{k}$$

and is calculated by the formula:

$$C\binom{n}{k} = \frac{n \times (n-1) \times (n-2) \times \cdots \times (n-k+1)}{k \times (k-1) \times (k-2) \times \cdots \times 1}$$

So the probability of matching exactly N of 4 selected numbers from a total 77 numbers is given by this formula:

$$\frac{C\binom{4}{N} \times C\binom{73}{4-N}}{C\binom{77}{4}}$$

To illustrate, let's calculate the probability of matching exactly 3 of 4 numbers. The above formula states that this probability is the number of ways to match 3 numbers from the 4 called numbers, multiplied by the number of ways to choose 1 number (4 minus 3) from the remaining 73 numbers, divided by the total number of ways to choose 4 numbers from the domain of 77 numbers. This yields a probability of 0.00021577. Using this same process, we calculate the probabilities shown in Figure 4 for matching 4 of 4, 3 of 4, 2 of 4, 1 of 4, and 0 of 4 numbers. Win for Life has no cases where matching 1 of 4 numbers produces a win, but we include it for completeness.

Numbers Matched	Probability
4	0.00000074
3	0.00021577
2	0.01165173
1	0.18383847
0	0.80429329

Figure 4 –Probability of Matching N out of 4 Numbers

With this information, we can now calculate the probability of winning the Win for Life prize, as well as the $50,000, $20,000, and $10,000 prizes. To win any of these prizes, the player must match all four numbers in one of the associated sets. The probability of matching all four numbers in any set is 0.00000074, so the probability of winning one of these awards is 0.00000074 multiplied by the number of sets associated with that award. Figure 5 summarizes the probabilities of winning these larger awards.

Prize	Matches Needed	Number of Sets	Match Probability	Win Probability
Win for Life	4	1	0.00000074	0.00000074
$50,000	4	2	0.00000074	0.00000148
$20,000	4	4	0.00000074	0.00000296
$10,000	4	8	0.00000074	0.00000591

Figure 5 – Probability of Winning Larger Awards

The probability of matching three or two numbers in any set is calculated the same way. That is, we take the probability of matching that many numbers (from Figure 4) and multiply that value by the number of sets. Since all 15 sets are eligible for these prizes, we multiply the match probability by 15 to get the win probabilities shown in Figure 6.

Prize	Matches Needed	Number of Sets	Match Probability	Win Probability
$25	3	15	0.00021577	0.00323659
$2	2	15	0.01165173	0.17477601

Figure 6 – Probability of Winning $25 or $2 Awards

Next, we need to calculate the probability of matching no numbers in any of the 15 sets to win the $3 prize. This is done differently from the probabilities we calculated for winning the other prizes because we must check for no matches in every set instead of matching some numbers in any set. Referring back to Figure 4, the probability of matching no numbers in one set is 0.80429329, which for simplicity we'll call P_0. It follows that the probability of matching no numbers in two sets is P_0 times P_0, the probability of matching no numbers in three sets is P_0 times P_0 times P_0, and so forth. So the probability of matching no numbers in all 15 sets is P_0 raised to the 15th power, or 0.03812560.

With these probabilities in hand, we can now calculate the individual RTP contribution of each prize. The RTP contribution is simply the amount won per unit wagered multiplied by the probability of winning that prize. The award amounts for all prizes except the top Win for Life prize are precisely defined as shown in the pay table in Figure 3, but the amount of the top prize varies based on how long the winner remains alive. For now, we'll use the 15-year duration assumed by the Oregon Lottery, so $1,000 per week ($52,000 per year) times 15 years gives an award amount of $780,000.

Since RTP is typically expressed as a percentage, we'll convert the probability values we just calculated to frequencies by multiplying the probabilities by 100. In addition, RTP calculations are simplified if the awards are expressed as the amount won per unit wagered, often called credits. For example, it costs $2 to play Win for Life, so the $50,000 prize is actually an award of 25,000 credits, where one credit equals $2. Figure 7 presents the RTP contribution for each prize, along with the overall Win for Life RTP, which is simply the sum of the individual RTP contributions.

Prize Description	Prize Amount	Award per Credit	Frequency	RTP Contribution
Match 4 in the WFL set	$780,000	390,000	0.000074%	28.82%
Match 4 in a $50K set	$50,000	25,000	0.000148%	3.69%
Match 4 in a $20K set	$20,000	10,000	0.000296%	2.96%
Match 4 in a $10K set	$10,000	5,000	0.000591%	2.96%
Match 3 in any set	$25	12.5	0.323659%	4.05%
Match 2 in any set	$2	1	17.477601%	17.48%
Match none in all sets	$3	1.5	3.812560%	5.72%
Total			21.614928%	65.67%

Figure 7 – Return to Player (RTP)

So if we use the expected 15-year duration for the top prize, the Win for Life RTP is 65.67%. This matches the 65% figure published by the Oregon Lottery.

Calculating the Hit Frequency

The hit frequency is the percentage of games where the player wins something, even if the amount won is less than the amount wagered. Analogous to RTP, a game with a hit frequency of 25% means that over the long haul, one can expect that 25% of the games played will result in a payout. It does not mean that a single player who plays 100 games will receive a payout in 25 of those games.

To calculate the Win for Life hit frequency, we can use the individual frequencies calculated when determining the RTP. We simply sum the frequencies associated with winning outcomes. Referring back to Figure 7, that sum is 21.61%.

Hit frequency can also be expressed as the odds of receiving a payout in a game, which is simply the reciprocal of the hit frequency percentage. This is 1 / 21.61, or 4.63, which matches the published Win for Life hit frequency.

Our calculated hit frequency and the published hit frequency represent the hit frequency for a single set of numbers on one ticket. Traditionally, hit frequency applies to the game in its entirety, not to components within the game (for a slot machine, this is analogous to the hit frequency for one pay line as opposed to the hit frequency for spins with all lines played). For example, consider the game ticket shown in Figure 2. If the called numbers for that drawing were 11-27-35-69, the last two $10,000 sets would each match two numbers (35 and 69), so this ticket would have two $2 prizes. For purposes of calculating the hit frequency, that should only count as a single $4 win. Simulation results show the actual (per ticket) hit frequency is a bit less than the published number and our calculations – 20.19%, or once per 4.95 games.

If there are no duplicate sets of four numbers on a given game ticket, it is impossible to win multiple prizes of $10,000 or more in that drawing (because winning those prizes requires matching all four numbers). Further, it is not possible to win multiple prizes on one ticket if the "match no numbers in all sets" prize is won. But as we've just shown, it is possible to have multiple "match two" or "match three" winners in conjunction with any prize except the "match none" prize in the same drawing.

Lifetime Prize Analysis

As mentioned earlier, the RTP published by the Oregon Lottery assumes a 15-year duration ($780,000 total) for the Win for Life top prize of $1,000 per week. It is not clear why lottery officials chose this number, but using demographics from the Oregon Lottery and the Oregon Department of Human Services, analysis reveals that lottery officials underestimated the prize duration.

According to the Oregon Lottery, the average age of a lottery player in Oregon is 47 years and 51% of the players are male (these figures are for all Oregon Lottery games and are not broken down for specific games). According to the Oregon Department of Human services, Oregon's statewide life expectancy is 77.2 years for males and 81.7 years for females. Using those figures, we can estimate the life expectancy of a typical lottery player in Oregon as 0.51 x 77.2 + 0.49 x 81.7, which is equal to 79.4 years. Subtracting the average age of a lottery player in Oregon (47) from this figure gives an estimated prize duration of 32.4 years, more than twice as long as estimated by lottery officials.

Our estimated 32.4-year duration for the top prize significantly changes the 65.76% Win for Life calculated RTP value. At 32.4 years, the average top prize amount becomes $1,684,800 (32.4 years times $52,000 per year), or 842,400 credits. This means the RTP contribution of the top prize becomes 62.25% and the overall RTP thus increases to 99.10%. But given how much of the RTP is tied to the top award, this really doesn't make the game that much more attractive.

Payout Limit and its Effect on the RTP

When previously discussing the pay table, we mentioned that if more than three players win the $1,000 per week for life prize in the same drawing, the prize is aggregated between those players as if there were three such winners. For example, suppose four players win the top prize in the same drawing. Each of those players would instead receive $1,000 x 3 divided by 4, or $750 per week for life.

Figure 5 shows that the probability of winning the top prize is 0.00000074. Because the prize aggregation starts with the fourth player to win the top prize in the same drawing, the probability of the prize being aggregated is 0.00000074 raised to the fourth power, or about once per 3.3×10^{24} games. To put that number in perspective, it's about thirty trillion times the estimated number of humans who have ever lived on this planet! Obviously, the chance of having to aggregate the top prize is so infinitesimal that we can ignore its effect on the overall RTP.

There is one caveat to the above discussion – it assumes the numbers in the Win for Life set are selected randomly, either by the game's quick pick option, or by players selecting their numbers independently of all other players. In reality, there will be some degree of dependency between player-selected numbers. For example, players might select birthday numbers more often, or several players who know each other might agree to select the same numbers for the same drawing. Such examples explain why the provision to aggregate the top award exists.

Player Simulation Metrics

To quantify the Win for Life player experience, a computer simulation of 100,000 players was performed. Each player wagered $2 per game, was given a $40 starting bankroll, and played 600 games or until their bankroll was exhausted, whichever occurred first. Figure 8 shows some of the overall simulation results.

Average games played	25.99
Hit frequency	20.19%
Players who hit the top prize	3

Figure 8 – Overall Simulation Results

The average number of games played (25.99) is substantially lower than the simulation metrics for most slot machines we design (typically between 120 and 160 average games played for the same simulation conditions). This can be explained by fact that the largest single component of the RTP is for the rarely occurring $1,000 per week top prize (28.82% if we use the Oregon Lottery's estimated top prize duration, 62.25% if we use ours). Taking this further, players who do not hit the top prize have an expected RTP of just 36.85% (the sum of the RTP values excluding the top prize). That applies to virtually all players; the simulation revealed that only 3 of the 100,000 simulated players hit the top prize.

Since each of the 100,000 simulated players had a $40 starting bankroll and wagered $2 per game, each player was guaranteed to play at least 20 games. Figure 9 shows the player time on device simulation results representing the percentage of players remaining at six different levels – from the 20-game minimum to the 600-game maximum. In our slot machine math models, our player simulations assume players play ten games per minute and we calculate time on device metrics at 2, 3, 5, 10, 20, and 60 minute thresholds. Using 20, 30, 50, 100, 200, and 600 games in the Win for Life player simulations allows us to directly compare the Win for Life time on device metrics to those of a typical slot machine.

Figure 9 shows that about 94% of the players were able to play more than 20 games. Or put another way, about 6% of the players won nothing and exhausted their bankroll within 20 games. Figure 9 also reveals a sharp drop in the percentage of players remaining between 20 and 30 games, and an even larger reduction between 30 and 50 games. This translates to over 99% of the players exhausting their starting bankroll before playing 50 games. If this were a slot machine, over 99% of the players would be broke within five minutes. That's far from desirable but typical of lottery-style games.

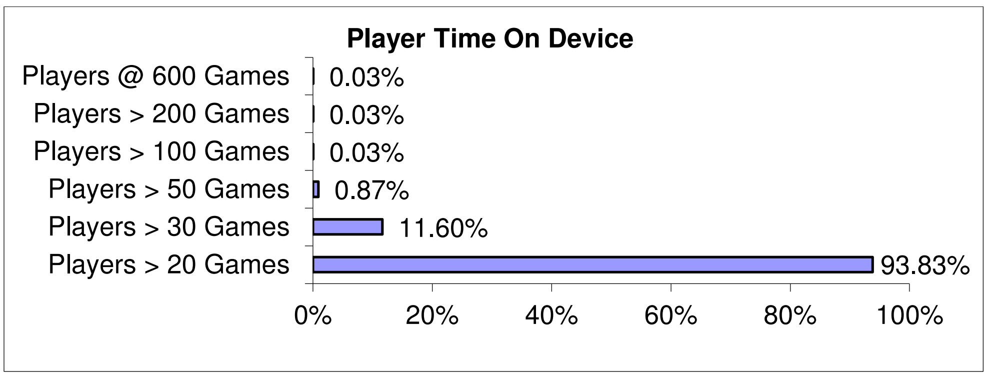

Figure 9 – Time on Device Simulation Results

Figure 10 shows the percentage of simulated players who won an award at or above selected thresholds (note that the percentages include awards *at or above* each threshold, so if we want to know what percentage of players within a specific threshold as their top award, we must subtract the percentage of players for the next threshold). Figure 10 shows that less than 10% of the simulated players won a prize of $5 or more across all drawings for a $2 wager. This isn't really surprising when you consider how much of the RTP is allocated to the top prize.

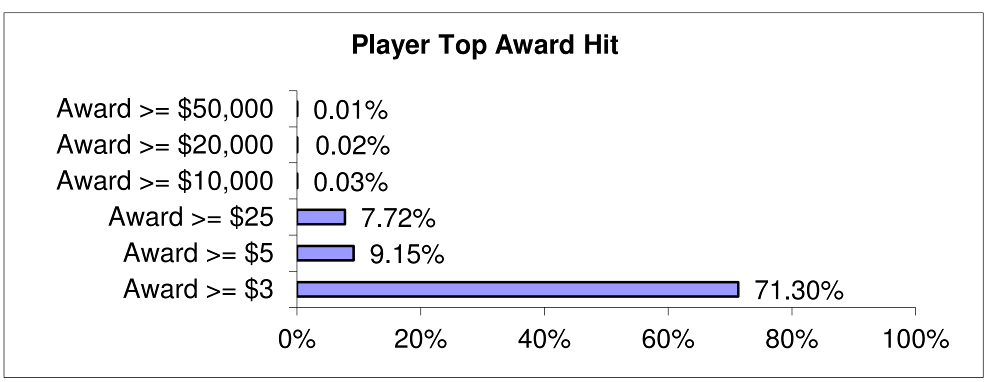

Figure 10 – Top Award Hit Simulation Results

Advantage Play Analysis

Advantage play (AP) is defined as a game state where the expected value (EV) of a given bet exceeds 100%. In other words, to determine if Win for Life provides AP opportunities, we need to determine if there is a pay table that provides an RTP above 100%. The top prize of $1,000 per week for life varies, but all other prize amounts are fixed, and collectively the fixed awards provide an RTP contribution of 36.85% (see Figure 7). So, we need to find the conditions when the top prize provides an RTP contribution of 100% minus 36.85%, or 63.15%.

The probability of winning the top prize is 0.00000074, so we need to find the per credit award N such that:

$$N \times 0.00000074 = 63.15\%$$

Using elementary algebra we calculate N = 63.15% divided by 0.00000074, or 853,378. That number is the required award per credit, so the Win for Life prize amount that satisfies the above equation for a $2 wager is twice that amount, or $1,706,656. At $52,000 per year, that would require the top award to have a payout duration of about 32.8 years. Recall that our lifetime prize analysis estimated the average top award prize duration at 32.4 years, and if we use that estimate, the overall Win for Life RTP is 99.10% – just below 100%, which confirms our AP calculations.

Figure 11 shows the Win for Life EV for various player ages based on the lifetime prize analysis presented earlier. The Win for Life prize is the only EV component dependent on the player age. These calculations include the minimum 5-year prize guarantee.

Player Age (years)	Duration (years)	WFL Prize	WFL EV	Game EV
18	61.4	$3,192,800	117.97%	154.81%
20	59.4	$3,088,800	114.12%	150.97%
25	54.4	$2,828,800	104.52%	141.37%
30	49.4	$2,568,800	94.91%	131.76%
35	44.4	$2,308,800	85.30%	122.15%
40	39.4	$2,048,800	75.70%	112.55%
45	34.4	$1,788,800	66.09%	102.94%
46	33.4	$1,736,800	64.17%	101.02%
47	32.4	$1,684,800	62.25%	99.10%
50	29.4	$1,528,800	56.49%	93.33%
55	24.4	$1,268,800	46.88%	83.73%
60	19.4	$1,008,800	37.27%	74.12%
65	14.4	$748,800	27.67%	64.51%
70	9.4	$488,800	18.06%	54.91%
75	5.0	$260,000	9.61%	46.45%
80	5.0	$260,000	9.61%	46.45%
85	5.0	$260,000	9.61%	46.45%
90	5.0	$260,000	9.61%	46.45%
95	5.0	$260,000	9.61%	46.45%
100	5.0	$260,000	9.61%	46.45%

Figure 11 – Expected Value for Various Player Ages

Because the top prize occurs so rarely and would have be paid out over 33 years before an AP condition exists, it is rather safe to say that Win for Life is not an appealing target for advantage players.

Conclusions

The calculations presented in this report show that Win for Life has a 65.67% RTP when using the Oregon Lottery's 15-year estimated top prize duration, but a 99.10% RTP when using our 32.4-year estimated top prize duration. The difference in RTP values is completely attributable to the difference between those two estimates. But no matter which estimate is used, the award distribution is greatly skewed toward seldom occurring high payouts.

Win for Life is technically vulnerable to AP conditions, but since the top prize occurs so rarely and is paid out over many years, the game really does not lend itself toward AP.

About the Author

Ted Gruber is president and co-founder of Ted Gruber Software, Inc. (TGS), a Nevada corporation specializing in the design and development of mathematical models for the gaming industry. TGS specializes in math for slot machines, bingo, keno, video poker, scratch cards, pull tabs, table games, side bets, skill games, gaming promotions, new games – you name it.

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